Generalising the TOV equilibrium condition

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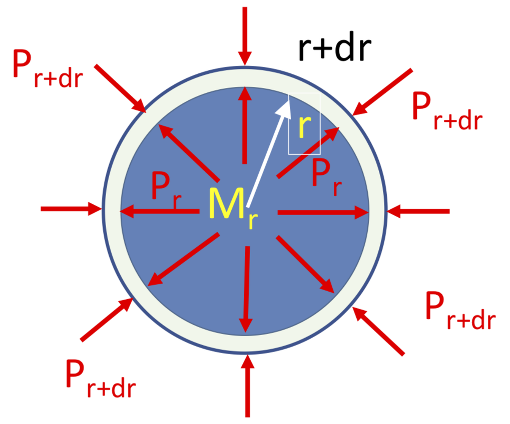
- **The Tolman-Oppenheimer-Volkov (TOV) equation appears as the relativistic counterpart of the classical condition for hydrostatic equilibrium.**

It applies to spherical systems [1], [2].



and establishes that static equilibrium requires the negative pressure gradient

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{\rho + P}{1-\frac{2GM}{r}}\left(\frac{M}{r^2} + 4\pi P r\right) . \tag{1}$$

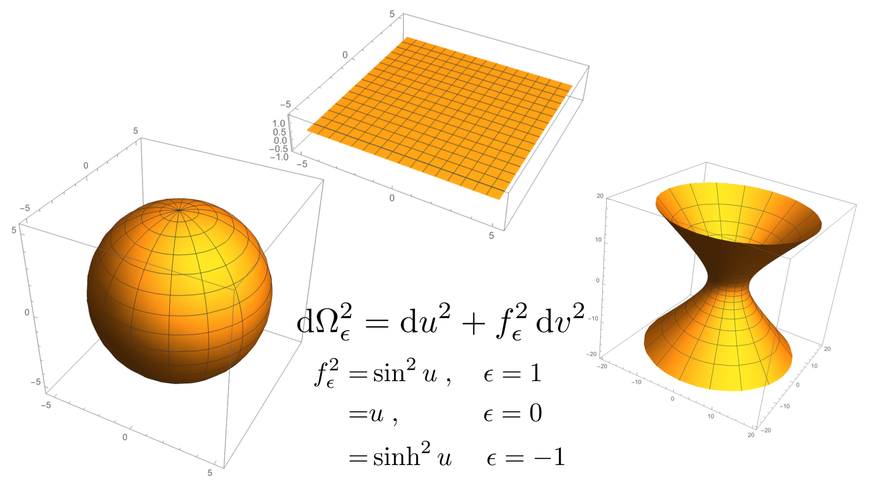
In the latter expression $(\rho + P)$ accounts for the relativistic inertia, $(1-2GM/r)^{-1}$ conveys the spatial curvature, and $M = M(r) = 4\pi \int \rho(r')\,\mathrm{d}r'$ is the Misner-Sharp-Hernandez (MSH) gravitational mass within the spherical shell of radial coordinate r.

- **In the present work we generalise TOV equation to the equilibrium of models endowed with other symmetries besides spherical.**

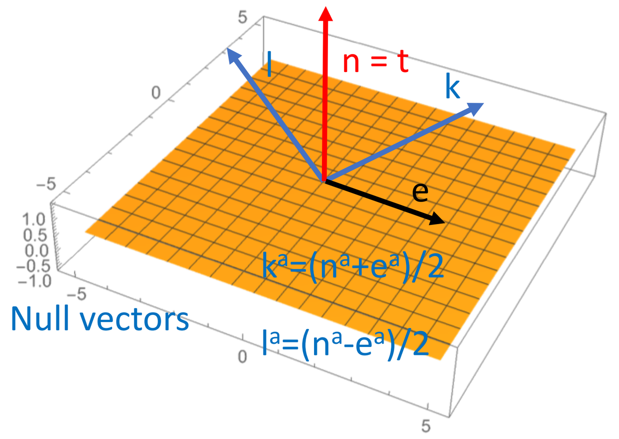
We envisage spacetime environments characterised by the line elements

$$\mathrm{d}s^2 = -\alpha^2(Y)\,\mathrm{d}T^2 + \frac{\mathrm{d}Y^2}{\epsilon - \frac{2\mu(Y)}{Y}} + Y^2\,\mathrm{d}\Omega_\epsilon^2 \tag{2}$$

where $\epsilon = 0, \pm 1$ distinguishes the 3 possible curvatures: $\epsilon = 0$ corresponds to flat spatial hypersurfaces, $\epsilon = +1$ corresponds to closed spatial hypersurfaces, and $\epsilon = -1$ corresponds to open spatial hypersurfaces, endowed with negative curvature



- **We apply the dual null formalism to spacetimes with a perfect fluid as the source.**



- **Assuming static solutions a (i.e., the existence of a Killing vector field orthogonal to the surfaces of symmetry).**

We obtain the generalised equation

$$\frac{\partial_Y P}{\rho + P} = -\left(\frac{\mu(Y)}{Y^2} + 4\pi P Y\right)\left(\epsilon - \frac{2\mu(Y)}{Y}\right)^{-1} , \tag{3}$$

which is what we call *the unified TOV equation*.

- **Definition of a quasi-local energy**

In order to treat the open geometries we have have put forward an extended definition of a quasi-local energy, since the Hawking-Hayward (as well as the MSH) definition only applies to compact foliations.

We propose the quasi-local mass-energy parameter $\mu(Y)$ by

$$M_\Sigma = \frac{\mu(Y)}{4\pi}\int S_\epsilon^2(\theta)\,\mathrm{d}\theta\mathrm{d}\phi\,. \tag{4}$$

(See details and discussion in Ref [1]).

The physics behind the models is deeply different!

- **Static solutions which are not spherically symmetric violate the weak energy condition (WEC)**
- **So open spacetimes tend to be endowed with a repulsive tension.**

A notorious example is provided by the cosmological constant Λ.

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- **Incompressible fluid solutions**

We illustrate our unified treatment obtaining analogs of Schwarzschild interior solution, for an incompressible fluid $\rho = \rho_0$ constant. (See Refs. [3] and [4]).

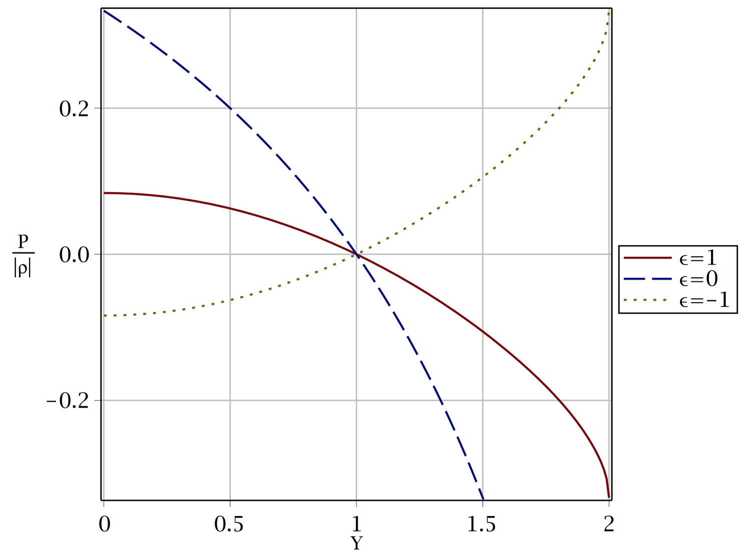

Figure 1. Pressure as function of Y for $Y_g = 1$ and $|Y_s| = 0.25$ for $\epsilon = 1$, $\epsilon = 0$ and $\epsilon = -1$.

Some popular models for modified gravity theories that aim to explain large scale phenomena, such as cosmological inflation and dark energy, also violate energy conditions, and there are several arguments in the literature suggesting those energy conditions should be abandoned as a criterion of viability of models.

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